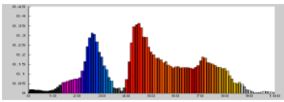
Dr. Qadri Hamarsheh



Signals and Systems
Lecture 3

Outline

- **Discrete-Time Signals-Introduction.**
- **Plotting discrete-time signals.**
- **Sampling Process.**
- **Discrete-Time Signal Representations**
- **Important D-T Signals**
- **Digital Signals**

Discrete-Time Signals-Introduction

The time variable t is said to be a discrete-time variable if t takes on only the discrete values $t = t_n$ for some range of integer values of n.

For example: $t = t_n = n$ for $n = 0,1,2,...,$

Discrete-time signal: is a signal that is a function of the discrete-time variable t_n; in other words, a discrete-time signal has defined values only at the discrete-time points $t = t_n$; so, a discrete time signal is a sequence of numbers indexed by integers.

Example: $x[n] \to n = ...,-3,-2,-1,0,1,2,3,...,$ **brackets** indicates D-T signal, **parenthesis** indicates C-T signal.

Plotting discrete-time signals

A **stem** plot emphasizes that the signal does not exist in-between integer n values. Sometimes we plot with line segments connecting the dots.

Matlab example1:

$x[n]$ is given by:

$$x[-3] = 2,\ x[-2] = -1,\ x[-1] = -3,\ x[0] = 5,\ x[1] = 2,\ x[2] = -1,\ x[3] = 7$$

with $x[n] = 0$ for all other n.

A plot of this signal (**see Figure 2_1**) can be generated by the following Matlab commands (see Chapter2_1.m):

```
% script Chapter 2_1.m
% Plot a discrete-time signal using Matlab
% we need two vectors to plot one-dimensional signal
% the first vector defines the horizontal axes:
% samples points to calculate the signal values.
% the second one defines the values of the signal
% at samples points (vertical axes)

n = -3:3; %first vector
%x[n]=0 for all other n.
x = [2,-1,-3,5,2,-1,7]; %second vector
stem(n,x,'filled');
xlabel('Time Samples: n');
```

```
ylabel('x[n]: Signal values');
title('Discrete-Time Signal');
axis([-4 4 -4 8]);
```

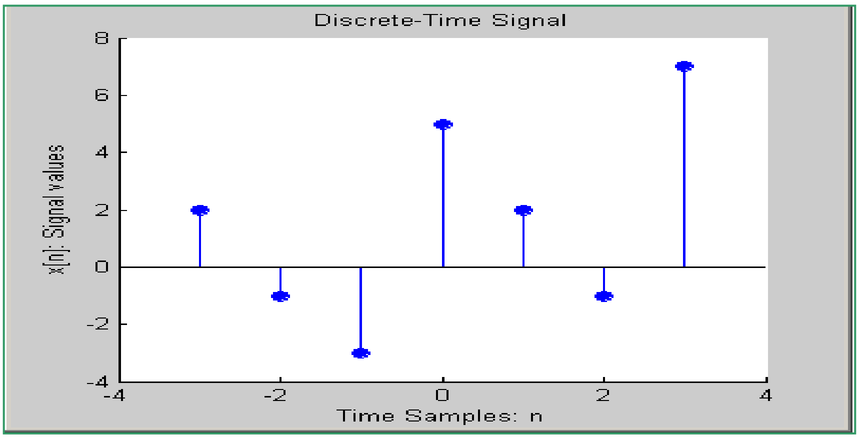


Figure 2_1

Sampling Process

one of the most common ways in which D-T signal arise is in sampling the C-T signals.

We can describe the sampling process as a **switch** that closes briefly every T seconds (as shown in figure 2_2) , the output of the switch can be viewed as a D-T signal that is a function of the discrete time points $t = t_n$, where $n = \ldots,-2,-1,0,1,2,\ldots,$

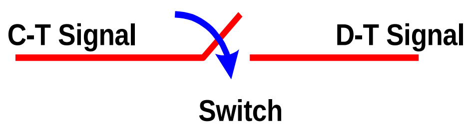


Figure 2_2: Sampling Process

The resulting D-T signal is called the **sampled version** of the original C-T signal, and T is called the **sampling interval**.

Sampling methods:

- ✓ **Uniform Sampling (T - constant)**
- ✓ **Nonuniform Sampling (T - variable)**

By definition of the sampling process, the value of $x[n]$ for any integer value of n is given by

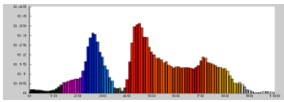
$$x[n] = x(t)\big|_{t=nT} = x(nT)$$

$\frac{1}{T}$ is called **sampling frequency or sampling rate** (F_s) in samples/seconds.

Important Question: How fast should we sample a specific signal?

We should sample a specific signal with sampling rate that is slightly more than **twice the highest frequency in this signal**.

Example: CD audio is sampled at **44100** samples per second $\Rightarrow T = \frac{1}{44100} \cong 22.69\ \mu\text{sec}$, because the humans can't hear frequencies above approximately **20 kHz**.



Discrete-Time Signal Representations

- ✓ **Graphical Representation**: as shown in figures 2_1.
- ✓ **Functional Representation**, such as

$$x[n] = \begin{cases} 2 & n = 1,4 \\ 3 & n = 2,3 \\ 0 & otherwise \end{cases}$$

- ✓ **Tabular Representation**, such as

n	...	-2	-1	0	1	2	3	4	...
$x[n]$	...	2	-1	5	7	4	-5	1	...

- ✓ **Sequence Representation**, such as

An infinite-duration signal or sequence with the time origin ($n = 0$) indicated by the symbol ↑ is represented as:

$$x[n] = \{\ldots,0,0,0,3,\underset{\uparrow}{6},3,1,-1,5,\ldots\}$$

A sequence $x[n]$ which is zero for $n < 0$, can be represented as

$$x[n] = \{\underset{\uparrow}{0},1,4,0,0,3,\ldots\}$$

Matlab example 2:

```
% script Chapter 2_2.m
% Plot a discrete-time signal
% x[n]=3*exp(-0.3n)sin(2n/3)(n-3)^2

n = 0:20; %x[n]=0 for all other n.
x = 3*exp(-0.3*n).*sin(2/3*n).*(n-3).^2;
stem(n,x,'filled');
xlabel('n');
ylabel('x[n]');
title('D-T Signal: x[n]=3exp(-0.3n)sin(2n/3)(n-3)^2');
axis auto;
```

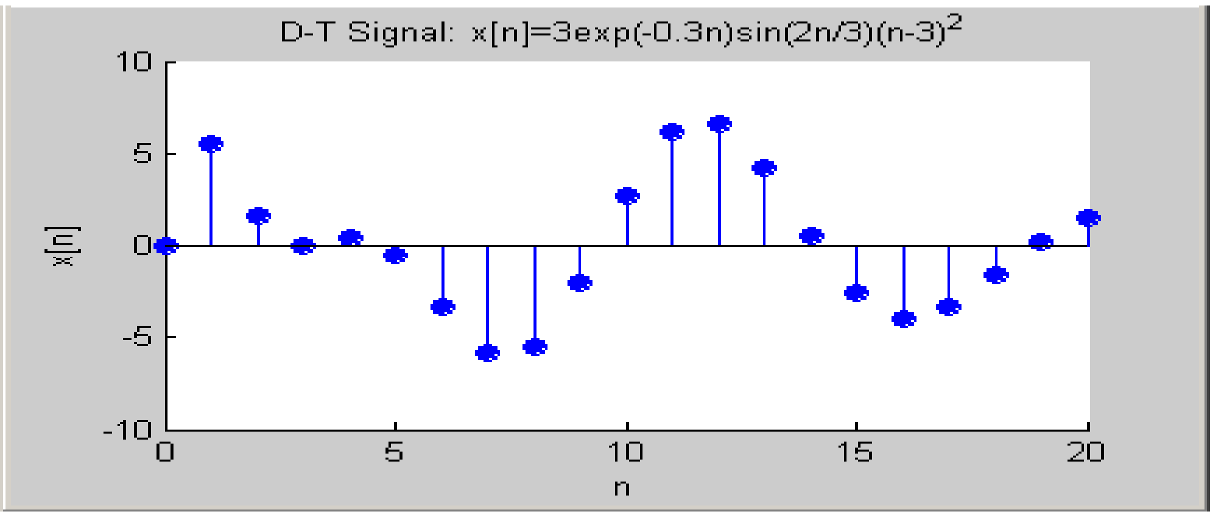
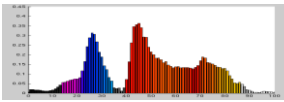


Figure 2_3



Important D-T Signals

- Much of what we learned about C-T signals carries over to D-T signals.

1. D-T Unit Step signal

D-T unit step signal $u[n]$ which is defined by

$$u[n]=\begin{cases}1, & n=0,1,2,3,\ldots \\ 0 & n=-1,-2,\ldots\end{cases}$$

D-T step signal can be obtained by **sampling** the C-T step $u(t)$ (sampled version of $u(t)$), the sketch of this signal is shown in figure 2_4 and the Matlab code to generate unit step signal is written in Chapter 2_3.m

```
%Script Chapter2_3.m
function unitstep(np)
% Generates and plots x[n] = u[n];
% -----------------------------------------
% UNITSTEP (NP)
%np - points' count
if np < 0
    error('argument np must satisfy np > 0')
end
n = [0:np];
x = [ones(1,np+1)];
stem(n,x,'filled');
xlabel('n');
ylabel('x[n]');
title('D-T Unit Step Signal')
axis ([-1 np+1 0 2]);
grid;
```

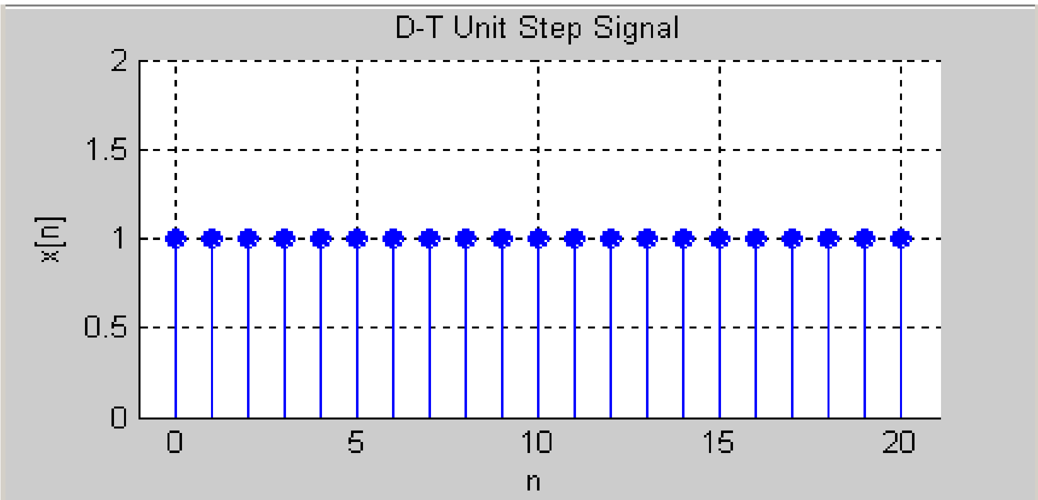


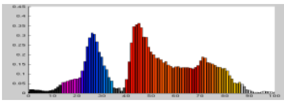
Figure 2_4

2. D-T Unit Ramp signal

D-T unit ramp signal $r[n]$ which is defined by

$$r[n]=\begin{cases}n & n=0,1,2,\ldots \\ 0 & n=-1,-2,\ldots\end{cases}$$

See figure 2_5a



If the unit ramp $r(t) = t \cdot u(t)$ is sampled, the result is given by

$$r[n] = r(t)\big|_{t=nT} = r(nT)$$

See figure 2_5b

These two signals in figure 2_5 are not the same. Unless the sampling interval T is equal to 1.

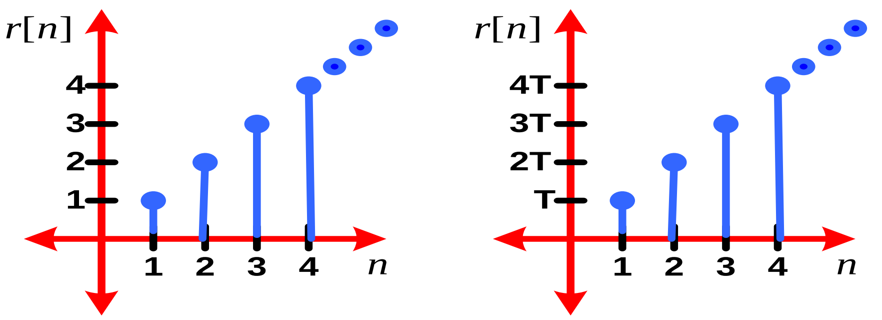


Figure 2_5a Figure 2_5b

Figure 2_5: D-T Unit-Ramp Signal

3. Unit Pulse (D-T Impulse)

There is no sampled version of the unit impulse $\delta(t)$ since $\delta(0)$ is not defined.
The unit-impulse signal, defined by

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

See figure 2_6.

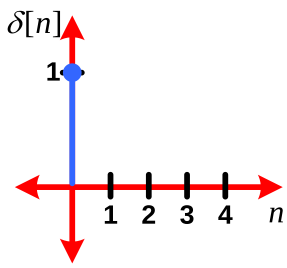


Figure 2_6: D-T Unit-Impulse Signal

Sifting property for D-T Delta function

Note:
$\delta[n]$ works inside summation , the same way $\delta(t)$ works inside integral

$$\sum_{n=-\infty}^{\infty} \delta[n] = 1 \xleftrightarrow{Compare} \int_{-\infty}^{\infty} \delta(t)dt = 1$$

$$\sum_{n=-\infty}^{\infty} x[n] \cdot \delta[n - n_0] = x[n_0] \xleftrightarrow{Compare} \int_{-\infty}^{\infty} x(t) \cdot \delta(t - t_0)dt = x(t_0)$$

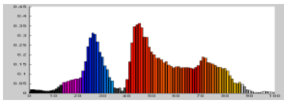
Any sequence can be expressed as:

$$x[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot \delta[n - k]$$

4. Periodic D-T signals (D-T Sinusoid)

A discrete signal $x[n]$ is periodic if there exists a positive integer r such that

$x[n + r] = x[n]$, for all integer n.



D-T sinusoid given by:

$$x[n] = A\cos(\Omega n + \theta)$$

-use upper case omega for frequency of D-T signals.

Ω – D-T frequency in radians per unit time T

what is the unit for Ω ?

$\Omega n + \theta$ must be in radians.

Ω is "how many radians jump for each sample ", Ω is in radians /sample

θ is the phase in radians.

The signal is **periodic** with period r if

$$A\cos(\Omega(n + r) + \theta) = A\cos(\Omega n + \theta)$$

Recall that cosine function repeats every 2π radians, so that

$$A\cos(\Omega n + \theta) = A\cos(\Omega n + 2\pi q + \theta) \text{ for all integer } q \text{, so}$$

$$\Omega r = 2\pi q \Rightarrow \Omega = \frac{2\pi q}{r} \text{ (Fundamental period).}$$

5. D-T Rectangular Pulse

Let S be a positive odd integer. D-T rectangular pulse signal $P_S[n]$ of length S defined by

$$P_S[n] = \begin{cases} 1 & n = -\frac{(S-1)}{2}, \ldots, -1, 0, 1, \ldots, \frac{(S-1)}{2} \\ 0 & all \quad other \quad n \end{cases}$$

A graphical representation of this signal is illustrated in **figure 2-7**.

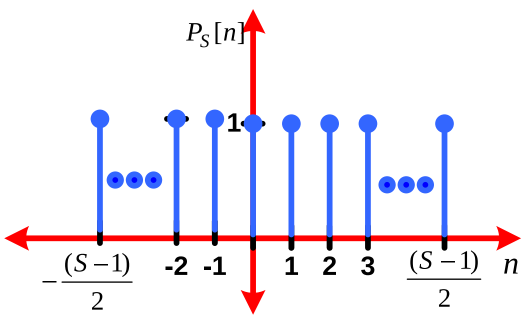


Figure 2_7: D-T Rectangular Signal

Digital Signals

A digital signal $x[n]$ is a discrete-time signal whose values belong to the finite set: $\{a_1, a_2, \ldots, a_N\}$, at each time instant t_n, we have

$$x(t_n) = x[n] = a_j \text{, for some } j \text{, where } 1 \le j \le N \text{.}$$

- ✓ A practical **ADC** not only gives a D-T signal but also one that is "**Digital**".
- ✓ **Binary signal** is a digital signal whose values are equal to 1 or 0:

$$x[n] = 0 \text{ or } 1\text{, for } n = \ldots, -2, -1, 0, 1, 2, \ldots$$

- ✓ The **sampled unit-step function and unit-pulse function** are examples of binary signals.